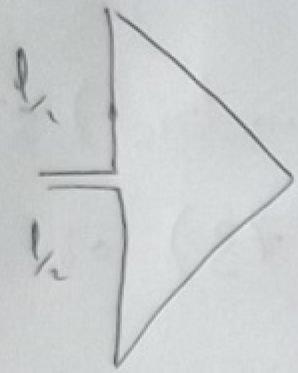
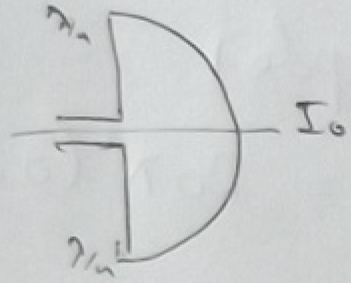
Sheet (4)

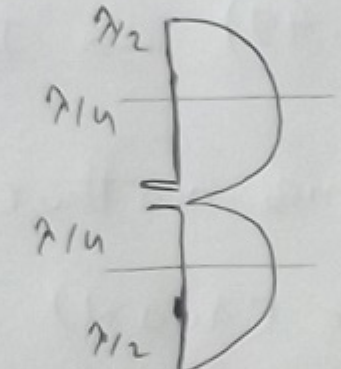
1. $l = 0.1\lambda$



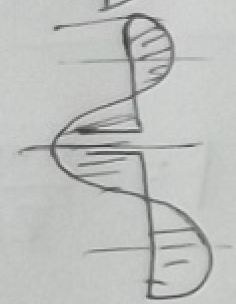
2. $l = \frac{\lambda}{2}$



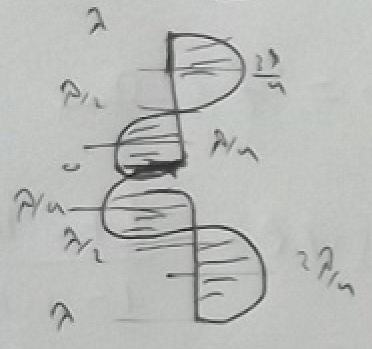
3. $l = \lambda$



4. $l = \frac{3}{2}\lambda$



5. $l = 2\lambda$



للتوضيح فقط

$0 \le \bar{z} \le l/2$

$$I(\bar{z}) = \begin{cases} I_0 \sin\left(\frac{kl}{2} - k\bar{z}\right) \\ I_0 \sin\left(\frac{kl}{2} + k\bar{z}\right) & -\frac{l}{2} \le \bar{z} \le 0 \end{cases}$$

① for $l = \frac{\lambda}{2}$ (مثلا) $\therefore \frac{kl}{2} = \frac{2\pi}{\lambda} \cdot \frac{\lambda/2}{2} = \frac{\pi}{2}$

(a) for $\bar{z} = 0 \rightarrow I(\bar{z}) = I_0$ (max)

(b) for $I(\bar{z}) = 0$ $\therefore \bar{z} = \pm\frac{\lambda}{4}$

(c) at $I = I_0$ $\therefore \frac{kl}{2} - k\bar{z} = \frac{\pi}{2}$ $\therefore \bar{z} = 0$

(مثلا)

② for $l = \lambda$

$$\frac{kl}{2} = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

(a) at $\bar{z} = 0 \rightarrow I(\bar{z}) = 0$

(b) at $I(\bar{z}) = 0 \rightarrow \pm\lambda/2$

(c) at $I = I_0 \rightarrow$

$\sin\left(\frac{kl}{2} - k\bar{z}\right) = 1$

$\frac{kl}{2} - k\bar{z} = \pi/2$

$\pi - k\bar{z} = \pi/2$

or $k\bar{z} = \pi/2$

$\therefore \bar{z} = \frac{\pi\lambda}{2 * 2\pi} = \lambda/4$

$\boxed{2}$ $P_{rad} = 100W = \frac{1}{2} I_o^2 R_r$

(a) for $\frac{\lambda}{2}$ dipole (R_r (standard) $\approx 73\,\Omega$)

$\therefore P_{rad} = 100 = \frac{1}{2} I_{o_1}^2 (73) \quad \therefore I_o = 1.654\,A$

(b) for 0.05λ (small dipole)

$R_r \cong 20\pi^2 \left(\frac{\ell}{\lambda}\right)^2 = 20\pi^2 (0.05)^2 \cong 0.49\,\Omega$

$\therefore P_{rad} = 100 = \frac{1}{2} I_{o_2}^2 (0.49) \rightarrow I_o \cong 20.14\,A$

* for small dipole, it takes 20 times as the current in $\frac{\lambda}{2}$ dipole to produce the same power (100 W)

$\boxed{3}$ $VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$, $\Gamma = \frac{Z_{in} - Z_o}{Z_{in} + Z_o}$

(1) for $\frac{\lambda}{2}$ dipole Assume $R_r = Z_{in} = 73\,\Omega$

$\therefore \Gamma = \frac{73-50}{73+50} = 0.187$, $VSWR = \frac{1+0.187}{1-0.187} = 1.46$

(2) for $\lambda/50$ dipole assume $Z_{in} = R_r = 20\pi^2 \left(\frac{\ell}{\lambda}\right)^2 = 0.316\,\Omega$

$\Gamma = \frac{0.316-50}{0.316+50} = -0.987$, $|\Gamma| = 0.987$

$\therefore VSWR = \frac{1+0.987}{1-0.987} = 152.8$

(4) Calc The 3dB Beamwidth → symmetrical Ant. of length 15m , $f_o = 10MHz$

Sol.

$$\lambda = \frac{c}{f} = \frac{3 * 10^8}{10 \times 10^6} = 30m \quad \therefore \frac{l}{\lambda} = \frac{1}{2} \quad \rightarrow \text{half wave dipole}$$

$$\therefore E_\theta = j60 \frac{I_o}{r} e^{-jkr} \left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right]$$

$$|E_\theta|_n = \frac{\left(\frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right)}{\left(\frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right)_{max}}$$

→ max occurs at $\pi/2$

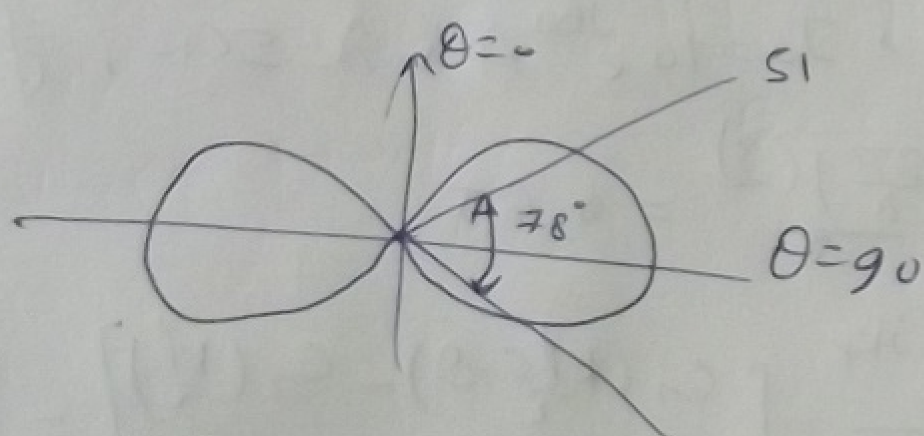
→ at 3dB Point

$|E_\theta|_n = \frac{1}{\sqrt{2}}$ or

$$\cos(\frac{\pi}{2}\cos\theta) = \frac{1}{\sqrt{2}} \sin\theta$$

by Try and error $\theta = 51°$

$\therefore$ HPBW $= 2|\theta_{max} - \theta_n| = 2|90 - 51| = 78°$



(5) antenna with max. linear dimensions (D). find Boundary of far field region so that max phase error doesn't exceed

a) $\pi/16$ rad b) $\pi/4$ rad c) 18° d) 15°

Sol

for $\frac{\pi}{8}$ max Phase error is

$$0.62\sqrt{D^3/\lambda} \leq r \leq 2D^2/\lambda$$

$$= \sqrt{(0.385)\frac{D^3}{\lambda}} \leq r \leq 2.D^2/\lambda$$

for any π/x $\sqrt{\left(\frac{x}{8}\right)(0.385)\frac{D^3}{\lambda}} \leq r \leq \left(\frac{x}{8}\right).2.D^2/\lambda$

let $\rightarrow \frac{\pi}{16}$ $\sqrt{(2)(0.385)\frac{D^3}{\lambda}} \leq r \leq (2)(2D^2/\lambda)$

let $\rightarrow \frac{\pi}{4}$ $\sqrt{\left(\frac{1}{2}\right)(0.385)\frac{D^3}{\lambda}} \leq r \leq \left(\frac{1}{2}\right)(2D^2/\lambda)$

let $\rightarrow 18^\circ \left(\frac{\pi}{10}\right)$ $\sqrt{\left(\frac{10}{8}\right)(0.385)\frac{D^3}{\lambda}} \leq r \leq \left(\frac{10}{8}\right)\left(\frac{2D^2}{\lambda}\right)$

let $15^\circ \left(\frac{\pi}{12}\right)$ $\sqrt{\left(\frac{12}{8}\right)(0.385)\frac{D^3}{\lambda}} \leq r \leq \left(\frac{12}{8}\right)\frac{2D^2}{\lambda}$

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6)

$l = 3$ Cm, $I_0 = 10e^{j60}$ A, $\lambda = 5$Cm, find E, H at 10 Cm (r), $\theta = 45^\circ$

Sol/ $\frac{kl}{2} = \frac{\left(\frac{2\pi}{\lambda}\right)(3)}{2} = 0.6\pi$

$$E_\theta = j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right] = \frac{j120\pi e^{-j\left(\frac{2\pi}{5\times10^{-2}}\right)(10\times10^{-2})}}{2\pi(10\times10^{-2})}\left[\frac{\cos(0.6\pi\cos45) - \cos(0.6\pi)}{\sin 45}\right]$$

$\Rightarrow E_\theta = 4620 e^{j11.52}$, $|E_\theta| = 4620$

$$H_\phi = \frac{|E_\theta|}{\eta} = \frac{4620}{120\pi} = 12.25A$$

$\boxed{7}$ $l = 5\lambda$, $r = 60\lambda$, find errors in phase & amplitude (far field).

sol

→ $\Delta\phi = \frac{K}{r}\left(\frac{z'^2}{2}\sin^2\theta\right)$, for max $\theta = 90°$ $z' = l/2$

$$\Delta\phi_{max} = \frac{2\pi}{\lambda}\cdot\frac{1}{60\lambda}\left(\frac{(2.5\lambda)^2}{2}\right) = 0.322 \text{ rad} = 18.7°$$

→ error in Amplitude $\frac{1}{R_2} - \frac{1}{R_1}$

$R_2 = 60\lambda$

$R_1 = \sqrt{r^2 + (-2rz'\cos\theta) + z'^2} = \sqrt{r^2 + z'^2} = \sqrt{(60\lambda)^2 + (2.5\lambda)^2} = 60.052\lambda$ ($\theta = 90$)

→ another solution for phase $\Delta\phi = K\Delta R = \frac{2\pi}{\lambda}(0.052\lambda) = 0.327 = 18.7°$

$\boxed{8}$ ~~far field~~ max Phase error $= 22.5°$ → occurs at direction $90°$ from axis along The largest dimension of antenna

- l_{max} of ant $= 5\lambda$, what do this max error reduces to at angle of $30°$ from axis along the length of antenna → use approximation term (1st higher Term)

sol far field $r = \frac{2l^2}{\lambda} = \frac{2(5\lambda)^2}{\lambda} = 50\lambda$

$\Delta\phi_e = \frac{K}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) = \frac{2\pi/\lambda}{50\lambda}\left(\frac{(2.5\lambda)^2}{2}\sin^2 30\right) = 0.0982$ rad $= 5.6°$

$z' \to 2.5\lambda$ $(l/2)$, $\theta \to 30°$

(Q) 15 MHz $\longrightarrow$ $E_0 = 0.05\,V/m$ $\longrightarrow$ 2.5λ, lossless

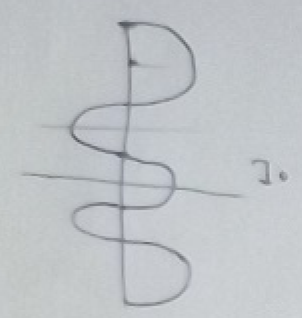
① plot current ② find θ for zero radiation strength

③ plot E (far field)

Sol/

ⓐ $2.5\lambda = \frac{5}{2}\lambda =$

موجتين ونص



ⓑ $|E_\theta| = \frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}$

$\frac{kl}{2} = \frac{2\pi}{\lambda}\cdot\frac{\frac{5}{2}\lambda}{2}$
$= 5\pi/2$
$\cos\left(\frac{5\pi}{2}\right) = \text{zero}$

$|E_\theta|_n = \frac{\cos\left(\frac{5\pi}{2}\cos\theta\right)}{\sin\theta}$

for nulls $\cos\left(\frac{5\pi}{2}\cos\theta\right) = 0$

at $\frac{5\pi}{2}\cos\theta = \pm(2n+1)\pi/2$ $n = 0, 1, 2 -$

at $n=0$ $\frac{5\pi}{2}\cos\theta = \pm\pi/2$

$\cos\theta = \pm\frac{1}{5}$, $\pm 78.46, \pm 101.54^\circ$

لو max
تكمل عن طريق جدول

θ	0	10	20 —	90
E_θ				

نعوض بالقيم E

$n=1$ $\frac{5\pi}{2}\cos\theta = \pm\frac{3\pi}{2}$, $\cos\theta = \pm\frac{3}{5}$

$\pm 53.13^\circ$, $\pm 126.87^\circ$

$n=2$ $\frac{5\pi}{2}\cos\theta = \pm\frac{5\pi}{2}$ $\cos\theta = \pm 1$

0, 180

$n=3$ $\frac{5\pi}{2}\cos\theta = \pm\frac{7\pi}{2}$ refused